**The powers of matrices In-class investigation**

**Solutions and marking key**

**Question 1 (a)**

|  |  |
| --- | --- |
| Solution  *M* = ,  , | |
| Marking keys/mathematical behaviours | Marks |
| * Identifies the expansion to power *n*. | 1 |

**Question 1 (b)**

|  |  |
| --- | --- |
| Solution | |
| Marking keys/mathematical behaviours | Marks |
| * Correctly expands powers 2,3,4 * Identifies the general solution | 1  1 |

**Question**  **2(a)**

|  |  |
| --- | --- |
| Solution  *M* =  , | |
| Marking keys/mathematical behaviours | Marks |
| * Tests with powers 2, 3 and a higher number * Generalises the rule | 1  1 |

**Question 2(b)**

|  |  |
| --- | --- |
| Solution  *M* =  , | |
| Marking keys/mathematical behaviours | Marks |
| * Tests with powers 2, 3 and a higher number * Generalises the rule | 1  1 |

**Question 3**

|  |  |
| --- | --- |
| Solution  *Let M* = | |
| Marking keys/mathematical behaviours | Marks |
| * Tests with at least two powers correctly * Identifies powers of 2 * Generalises the rule | 1  1  1 |

**Question 4 (a)**

|  |  |
| --- | --- |
| Solution  *M* =  , | |
| Marking keys/mathematical behaviours | Marks |
| * Tests with powers 2, 3 and a higher number * Generalises the rule | 1  1 |

**Question 4 (b)**

|  |  |
| --- | --- |
| Solution | |
| Marking keys/mathematical behaviours | Marks |
| * Correctly identifies the power of *M*. | 1 |

**Question**  **5 (a)**

|  |  |
| --- | --- |
| Solution  *M* =    NB Powers of 2 and 3 add to *n* if *n* is either even or odd. | |
| Marking keys/mathematical behaviours | Marks |
| * Tests correctly at least 4 powers of *M* * Identifies that there are different rules for odd and even powers * Correctly expands 6 matrices when *n* is even * Has the correct powers of 2 and 3 in correct location when *n* is odd | 1  1  2  2 |

**Question**  **5 (b)**

|  |  |
| --- | --- |
| Solution  *M* = | |
| Marking keys/mathematical behaviours | Marks |
| * Recognises 6 is even so chooses correct rule * Predicts correct signs and powers | 1  1 |

**Question**  **6**

|  |  |
| --- | --- |
| Solution | |
| Marking keys/mathematical behaviours | Marks |
| * Determine the inverse matrix *S-1*. * Shows why  by taking out the negative. | 1  1 |

**Question**  **7**

|  |  |
| --- | --- |
| Solution **for the even rule**  If *n* is an EVEN number, then | |
| Marking keys/mathematical behaviours | Marks |
| * Develops the pattern then explains the coefficient in terms of * Shows that the matrix  to the power  gives the correct matrix for *n* = 1, 2, 3 and 4 * Suggests the pattern is cyclic | 2  4  1 |

**Question**  **7**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Solution **for the odd rule**  If *n* is an odd number, then  The coefficients are   |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | Power *n* | 1 | 3 | 5 | 7 | 9 | | Coefficients from the table | 1 | 2 | 4 | 8 | 16 | |  | 20 | 21 | 22 | 23 | 24 |   The coefficients are equal to powers of 2. The power of the two is equal to .  Therefore the coefficients are equal to  Consider   |  |  | | --- | --- | | Power *n* | 1 | |  | 0 | |  |  |  |  |  | | --- | --- | | 3 | 5 | | 1 | 2 | |  |  |  |  |  | | --- | --- | | 7 | 9 | | 3 | 4 | |  |  |   The cycle repeats. The matrices are the same as the matrices in the table for the given *n* values. Consider the product of the coefficients and the matrices. They are the same.  Therefore the rule works for odd *n*. | |
| Marking keys/mathematical behaviours | Marks |
| * Develops the pattern then explains the coefficient in terms of n * Shows that the product of the matrix  and the matrix  to the power  gives the correct matrix for *n* = 1, 2, 3 and 4 * States the pattern is cyclic | 2  4  1 |